# VerSiS: Verification and Simulation of embedded hybrid Systems

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# Outline

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# Overview

## Heterogenous embedded hybrid systems

- Digital hardware and software components
- Analog sensor and actuator components

## Goals

- Extension of formal verification methods to analog components
- Modeling, simulation, and automated approximation of multi-physical systems
- Digital verification with algebraic methods
- Integration in work flow for design and analysis of behavior-based robotic systems as sample application

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# Scientific Personnel and Experience

## Adaptive Systems (Fraunhofer ITWM)

- Dr. Patrick Lang
- Dr. Alexander Dreyer
- Dr. Jochen Broz (sponsored)

- Analog Insydes: Tools for symbolic/numeric analysis of analog circuits, based on Differential-Algebraic Equations (DAEs)
- Modeling and simulation of mechatronic systems
- Symbolic approximation methods and tolerance analysis

# Scientific Personell and Experience (cont'd)

## Reactive Systems Group (Computer Science)

- Prof. Klaus Schneider (chair)
- Dr. Raffaella Gentilini (sponsored)
- Jens Brandt (sponsored)

- Averest Tool: specification, verification, and implementation of reactive systems
- Model checking, automated theorem proving, and formal methods for systems' analysis and verification
- Temporal logics and automata theory

# Scientific Personell and Experience (cont'd)

## Robotics Research Lab (Computer Science)

- Prof. Karsten Berns
- Martin Proetzsch (sponsored)

- Realization of autonomous mobile robots, like wheel-based vehicles (indoor, outdoor), climbing robots, as well as humanoid assistance systems
- Modular Controller Architecture (MCA) for implementing complex systems featuring real-time analysis and state monitoring
- Biologically motivated behavior-based system design approach

# Scientific Personell and Experience (cont'd)

## Algebra, Geometry and Computer Algebra (Mathematics)

- Prof. Gert-Martin Greuel (chair)
- Oliver Wienand

- SINGULAR: Fast Gröbner system for polynomial systems
- POLYBORI: Gröbner approaches for Boolean equation systems for digital verification (DFG project together with ITWM-AS and MFO)

# Additional Information

Project start: 1.10.2005

## External Cooperations

- Michael Brickenstein
  (Mathematisches Forschungsinstitut Oberwolfach *MFO*)
- Dr.-Ing. Markus Wedler, Evgeny Pavlenko (EIT, TU KL)

# Outline

# Development of Robot Control Systems


RA

Robust Autonomous Vehicle for Outdoor Navigation

- Complexity of solving tasks especially in outdoor robotics
- Approach: decompose control system into *behaviors* inspired by observations in biology
- Problem: assure certain safety critical properties of the system

⇒ Need for analysis methods for behavior-based networks

# Development of Robot Control Systems


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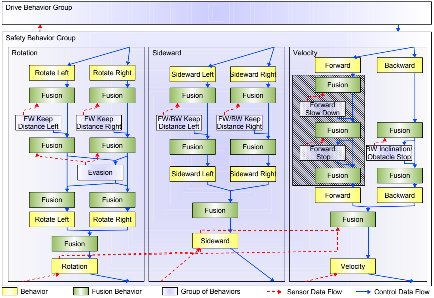
$\Rightarrow$ Need for analysis methods for behavior-based networks

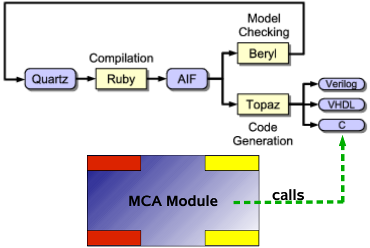
# Safety Behavior Network for RAVON




## Control Structure

- MCA: Modular Controller Architecture
- Behaviors for assuring vehicle integrity, uniform interaction
- Slope observation, obstacle proximity (several sensors)
- Choose behaviors influencing vehicle velocity for verification

# Verification of Safety Behaviors: Tools




## Averest

- Tools for verifying temporal properties of synchronous systems
- Quartz compiler: `ruby`
- Symbolic model checker: `beryl`
- Generate code in software and hardware: `topaz`

## Interaction with MCA

- MCA module calls provided C-methods

# Verification: Specifications and Performance

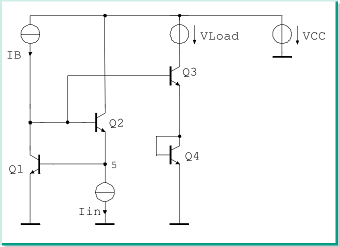
- Verified 15 specifications concerning safety properties of the vehicle
- Examples:

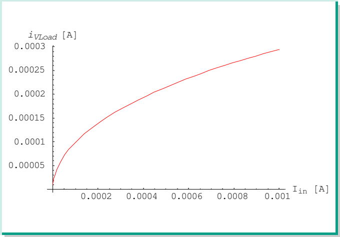
```
A G (co_velocity <= ci_velocity);
A G ((si_camera_distance >= MAX_VEL_OBSTACLE_DIST)
   & (si_scanner_distance >= MAX_VEL_OBSTACLE_DIST)
   & (si_roll <= MAX_VELOCITY_INCLINATION)
   & (si_pitch <= MAX_VELOCITY_INCLINATION)
   -> co_velocity == ci_velocity);
A G (si_roll >= STOP_INCLINATION
 -> co_velocity == 0u);
```

- Uniform modular structure of behavior-based architecture proven favorable for verification
- Verification runtime: 8.85 s for 8 bits

# Outline

# Symbolic Approximation using Simulations




## Symbolic equations

- 27 equations
- 17 parameters

$$i_{CS,Q2} + i_{VDC} + i_{VLoad} = -I_B$$
$$i_{BS,Q2} + i_{BS,Q3} + i_{CS,Q1} = I_B$$
$$i_{BS,Q4} + i_{CS,Q4} + i_{ES,Q3} = 0$$
$$i_{BS,Q1} + i_{ES,Q2} = -I_{in}$$
$$i_{CS,Q3} - i_{VLoad} = 0$$
$$-i_{BS,Q1} - i_{CS,Q1} - i_{ES,Q1} = 0$$
$$i_{BS,Q1} = ib_{Q1}$$
$$-i_{BS,Q1} - i_{ES,Q1} = ic_{Q1}$$
$$ic_{Q1} = v_5 G_{min} + \left(-1 + e^{\frac{v_5}{Vt}}\right) Is_{Q1} - \left(1 + \frac{1}{Br_{Q1}}\right) \left((-v_3 + v_5) G_{min} + \left(-1 + e^{\frac{-v_3+v_5}{Vt}}\right) Is_{Q1}\right)$$
$$ib_{Q1} = \frac{v_5 G_{min} + \left(-1 + e^{\frac{v_5}{Vt}}\right) Is_{Q1}}{Bf_{Q1}} + \frac{(-v_3+v_5) G_{min} + \left(-1 + e^{\frac{-v_3+v_5}{Vt}}\right) Is_{Q1}}{Br_{Q1}}$$
$$-i_{BS,Q2} - i_{CS,Q2} - i_{ES,Q2} = 0$$
$$i_{BS,Q2} = ib_{Q2}$$
$$-i_{BS,Q2} - i_{ES,Q2} = ic_{Q2}$$
$$ic_{Q2} = (v_3 - v_5) G_{min} + \left(-1 + e^{\frac{v_3-v_5}{Vt}}\right) Is_{Q2} - \left(1 + \frac{1}{Br_{Q2}}\right) \left((-v_1 + v_3) G_{min} + \left(-1 + e^{\frac{-v_1+v_3}{Vt}}\right) Is_{Q2}\right)$$
$$ib_{Q2} = \frac{(-v_1+v_3) G_{min} + \left(-1 + e^{\frac{-v_1+v_3}{Vt}}\right) Is_{Q2}}{Br_{Q2}} + \frac{(v_3-v_5) G_{min} + \left(-1 + e^{\frac{v_3-v_5}{Vt}}\right) Is_{Q2}}{Bf_{Q2}}$$
$$-i_{BS,Q3} - i_{CS,Q3} - i_{ES,Q3} = 0$$
$$i_{BS,Q3} = ib_{Q3}$$
$$-i_{BS,Q3} - i_{ES,Q3} = ic_{Q3}$$
$$ic_{Q3} = (v_3 - v_4) G_{min} + \left(-1 + e^{\frac{v_3-v_4}{Vt}}\right) Is_{Q3} - \left(1 + \frac{1}{Br_{Q3}}\right) \left((v_3 - v_{out}) G_{min} + \left(-1 + e^{\frac{v_3-v_{out}}{Vt}}\right) Is_{Q3}\right)$$
$$ib_{Q3} = \frac{(v_3-v_4) G_{min} + \left(-1 + e^{\frac{v_3-v_4}{Vt}}\right) Is_{Q3}}{Bf_{Q3}} + \frac{(v_3-v_{out}) G_{min} + \left(-1 + e^{\frac{v_3-v_{out}}{Vt}}\right) Is_{Q3}}{Br_{Q3}}$$
$$-i_{BS,Q4} - i_{CS,Q4} - i_{ES,Q4} = 0$$
$$i_{BS,Q4} = ib_{Q4}$$
$$-i_{BS,Q4} - i_{ES,Q4} = ic_{Q4}$$
$$ic_{Q4} = v_4 G_{min} + \left(-1 + e^{\frac{v_4}{Vt}}\right) Is_{Q4}$$
$$ib_{Q4} = \frac{v_4 G_{min} + \left(-1 + e^{\frac{v_4}{Vt}}\right) Is_{Q4}}{Bf_{Q4}}$$
$$v_1 - v_{out} = V_{Load}$$
$$v_1 = V_{DC}$$

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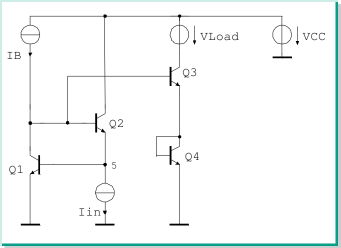
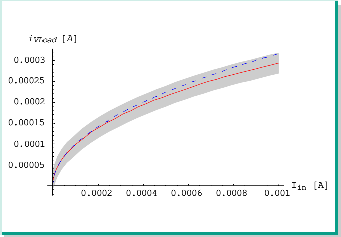



## Symbolic equations

- 27 equations
- 17 parameters

## Reduced equations

- 4 equations
- 7 parameters

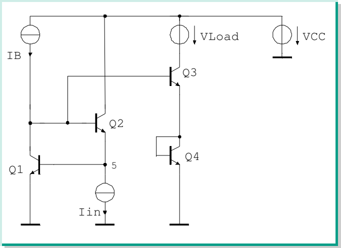
$$-I_{in} + e^{\frac{v_3 - v_5}{Vt}} \, Is_{Q2} = 0$$

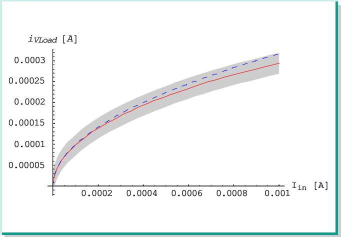
$$I_B - e^{\frac{v_5}{Vt}} \, Is_{Q1} = 0$$

$$i_{VLoad} - e^{\frac{v_3 - v_4}{Vt}} \, Is_{Q3} = 0$$

$$i_{VLoad} - e^{\frac{v_4}{Vt}} \, Is_{Q4} = 0$$

# Symbolic Approximation using Simulations

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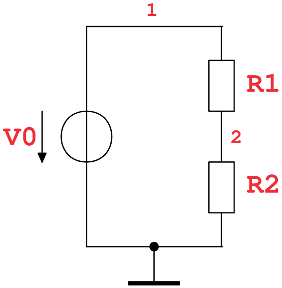
## Explicit formula

$$i_{vload}^{simple} = \sqrt{\frac{Is_{Q3}\, Is_{Q4}}{Is_{Q1}\, Is_{Q2}}}\sqrt{I_B}\sqrt{I_{in}}$$

# Parameter Tolerances using Interval-arithmetic




- Simulations ensure validity for discrete settings only

**V0** $= 1\,\mathrm{V}$
**R1** $= 10\Omega$
**R2** $= 100\Omega$

$$V_2 = \frac{V_0}{R_1/R_2+1} \approx 0.909\,\mathrm{V}$$

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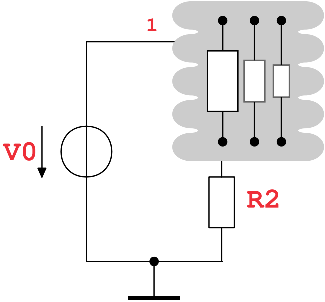


- Simulations ensure validity for discrete settings only
- Interval-valued computations can verify properties for domains

$$\texttt{V0} = 1\,\mathrm{V} \pm 10\%$$
$$\texttt{R1} = 10\Omega \pm 10\%$$
$$\texttt{R2} = 100\Omega \pm 10\%$$

$$V_2 = \frac{V_0}{R_1/R_2+1} \approx 0.909\,\mathrm{V}$$
$$V_2 \in \frac{[0.9,1.1]}{[9,11]/[90,110]+1} = [0.80, 1.02]$$

# Validation of Symbolic Approximations

## Compare different accuracy levels

$$i_{vload} \approx i_{vload}^{simple} \ ?$$

$$i_{vload} = \frac{\sqrt{Is_{Q3}\, Is_{Q4}\, ((0.025\, Is_{Q3}\, Is_{Q4} - Is_{Q2}\, Is_{Q1})10^{-2}\, I_{in}{}^{2} + Is_{Q1}\, Is_{Q2}\, I_B\, I_{in})} - 5\cdot 10^{-3}\, I_{in}\, Is_{Q3}\, Is_{Q4}}{Is_{Q1}\, Is_{Q2}}$$

$$i_{vload}^{simple} = \sqrt{\frac{Is_{Q3}\, Is_{Q4}}{Is_{Q1}\, Is_{Q2}}}\, \sqrt{I_B}\, \sqrt{I_{in}}$$

## Example

For $Is_{Qi} = 10^{-16}\mathrm{A}$, $I_{in}/1\mathrm{mA} \in [0.2, 0.3]$, and $I_B/1\mathrm{mA} \in [0.1, 0.2]$ interval-techniques **prove** $|i_{vload} - i_{vload}^{simple}| < 2.5\%$

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# Formal Verification of Analog Circuits

**Motivation:**
**Simulation vs. Formal Verification**

Contrary to simulation, formal verification techniques aim at:

- Verifying system properties before manufacturing
- Making the verification process automatic
- Coping with the variability of parameters and input signals

# Formal Verification of Analog Circuits

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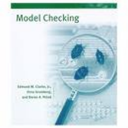
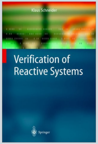
## The Model Checking Verification Technique

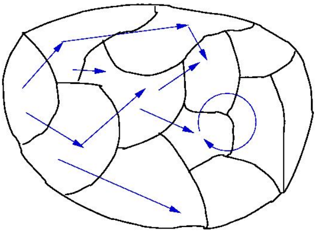



A well established technique for the verification of digital finite systems. Properties stated via a temporal logic language verified:

- completely automatically
- on any computational path of the graph-modeled system

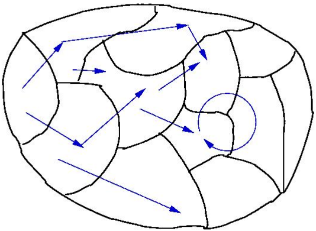
# Formal Verification of Analog Circuits



## The Model Checking Verification Technique

A natural way of extending the model checking technique to analog circuits involves the extraction of some discrete approximation of the infinite graph underlying the evolution of an analog circuit.

# Formal Verification of Analog Circuits

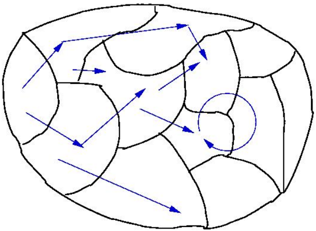


## The Model Checking Verification Technique

However, two major issues have to be considered to obtain an effective final verification technique:

- Need of adequate languages to state analog properties.
- Need of formal tools to ensure preservation results from the abstraction to the analog model.

# Formal Verification of Analog Circuits



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# $CTL_f$: a Language for the Modeling of Analog Prop.

$CTL_f$ is an enrichment of classical CTL language, where we allow 'basic formulæ' to state the membership in boxes of values given by arbitrary elementary functional relations.

## The $CTL_f$ Syntax

Let $X = \{x_1, \ldots, x_n\}$ be a finite set of real valued variables. The set of $CTL_f$ formulæ is defined according the following grammar:

| | | |
|---|---|---|
| $\phi ::=$ | $f(x_{i_1}, \ldots, x_{i_m}) \triangleright I$ | (basic formulæ testing membership in box $I$) |
| | $\neg\phi \mid \phi \vee \phi$ | (boolean combinators) |
| | $\mathrm{E}\phi\mathrm{U}\phi \mid \mathrm{A}\phi\mathrm{U}\phi$ | (temporal combinators & path quantifiers) |

where $1 \leq i_1 \leq \cdots \leq i_m \leq n$, $f : \mathbb{R}^m \mapsto \mathbb{R}^p$ is an arbitrary composition of elementary functions, $I$ is a box in $\mathbb{IR}^p$.

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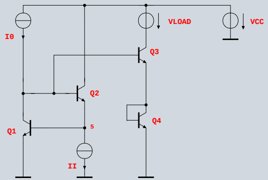
# CTL$_f$: a Language for the Modeling of Analog Prop.

## Example




- In the *square root function block* property (1) holds:

$$I_0 = \beta \sqrt{I_I} \sqrt{I_B} \ , \qquad (1)$$

- CTL$_f$ formula encoding property (1):

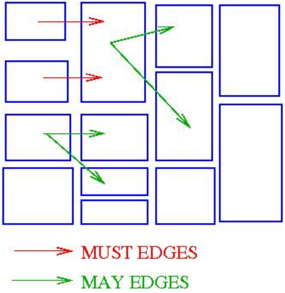
$$\mathrm{AG}((I_0 - \beta \sqrt{I_I} \sqrt{I_B}\ ) \triangleright [-\delta, +\delta]) \qquad (2)$$

# 3-Valued $CTL_f$ Model Checking on Analog Circuits

We use Interval Arithmetic to provide a three-valued semantics to our $CTL_f$ logic on suitable abstraction of the states-space of analog circuits (box-modal abstractions).

**The Three-Valued Semantics**

A natural choice since the value of our $CTL_f$ formulae could be indefinite on abstract states (boxes)



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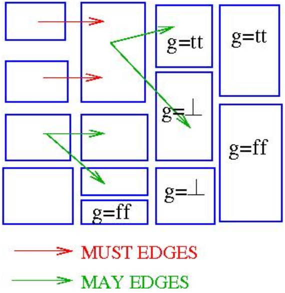

# 3-Valued $CTL_f$ Model Checking on Analog Circuits

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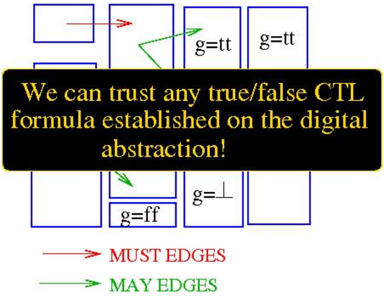
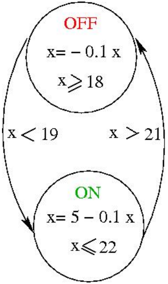
## The Three-Valued Semantics

Allow to apply 3-valued model checking techniques and to state a fundamental preservation result for $CTL_f$ verified formulae:



# Outline

# The Hybrid Automata (HA) Model




## Hybrid Automata

- State-of-the-art formalism for modeling hybrid systems (interacting continuous/discrete dynamics) in CS.
- Combine the automata tool (for modelling the discrete component) with systems of differential equations (for representing continuous dynamics).
- Inverse relation between decidability and modeling capability.

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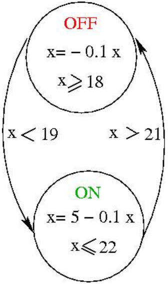


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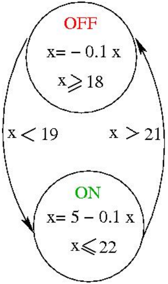


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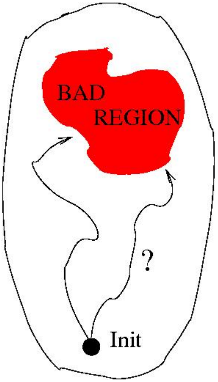
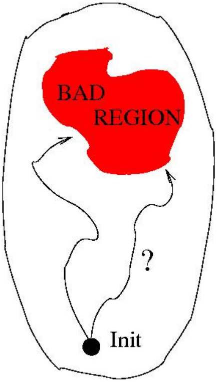
# Hybrid Automata Verification




## The Reachability Problem

- Is the region *B* reachable from any initial state?
- Safety analysis can be reduced to reachability.
- Most (interesting) classes of HA known undecidable with respect to reachability.

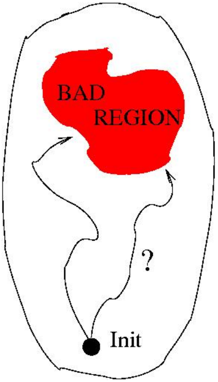
# Hybrid Automata Verification




## Reachablity Analysis over Undecidable Hybrid Automata

- Known symbolic techniques force to choose between reach-sets over-/under-approx. (i.e between trying to prove/disprove safety).
- Mostly based on the simulation preorder that in general allows for the preservation of only true prop. in the universal fragment of CTL.

# Hybrid Automata Verification




## Reachablity Analysis over Undecidable Hybrid Automata

- In a recent joint work, we introduce the first framework for combining over-/under approximated reach-set analysis on (undecidable) HA.
- Our method is based on a succession of abstractions, where general CTL stated reactive system properties can be monotonically proved/disproved.

# Outline

# More on VerSiS Project: Ongoing and Future Work

- Extension of verified behavior region in robot verification
- Optimization of the behavior architecture using results from model checking and robot verification
- Expand combined interval and three-valued logic approach to real-world applications
- Ongoing work on the use of computer algebra methods (in particular Gröbner Bases methods) for the analysis of hybrid automata.
- Ongoing work related to the issue of modeling hybrid systems by means of synchronous languages, in order to integrate them into the *Averest Tool*.

# Further Reading


- Raffaella Gentilini, Klaus Schneider, and Alexander Dreyer.
Three-valued automated reasoning on analog properties.
In *17th ACM Great Lakes Symposium on VLSI (GLSVLSI '07). Stresa-Lago Maggiore, Italy*. ACM Press, March 2007.

- Raffaella Gentilini, Klaus Schneider, and Bud Mishra.
Series of abstractions of hybrid automata for monotonic ctl model checking.
In *International Symposium on Logical Foundations of Computer Science (LFCS '07). New York, U.S.A.*, LNCS. Springer, 2007.

- Alexander Dreyer.
Interval methods for analog circuits.
Technical report, Fraunhofer ITWM, 2006.

# Further Reading (cont'd)

Michael Brickenstein and Alexander Dreyer.
Polybori: A framework for Gröbner basis computations with Boolean polynomials.
Submitted to: *Effective Methods in Algebraic Geometry MEGA 2007*, June 2007.

Martin Proetzsch, Tobias Schüle, Karsten Berns, and Klaus Schneider.
Formal Verification of Safety Behaviours of the Outdoor Robot RAVON.
Submitted to: *4th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2007). Angers, France*. May 2007.